Shortest Paths

- Usually, the highway structure can be represented by graphs with vertices representing cities and edges representing sections of highways.
- Edges may be assigned weights to represent the distance or the average driving time between two cities connected by a highway.
- Often, for most drivers, it is desirable to find the shortest path from the originating city to the destination city.

Single-Source Shortest Paths

- **Given:** A single source vertex in a weighted, directed graph.
- Want to compute a shortest path for each possible destination.
  - Similar to BFS.
- We will assume either
  - no negative-weight edges, or
  - no reachable negative-weight cycles.
- Algorithm will compute a shortest-path tree.

Dijkstra's Algorithm

Assumes **no negative-weight edges**.

Maintains a set S of vertices whose SP from s has been determined

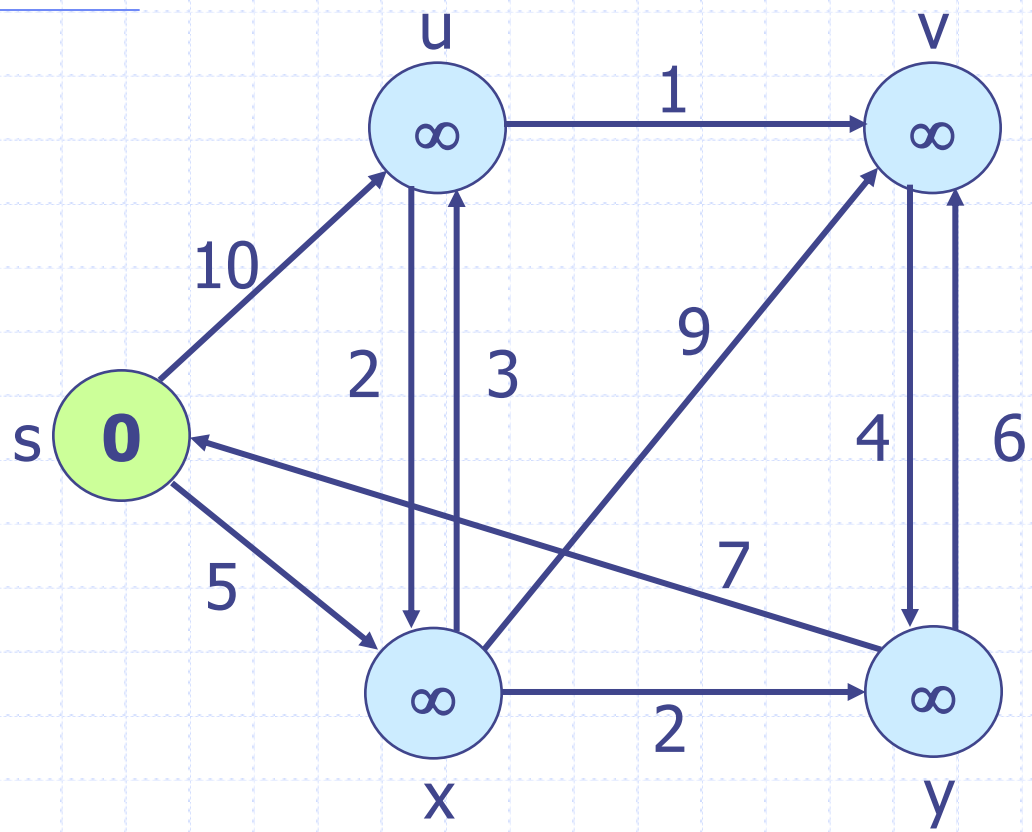
epeatedly selects u in V–S with minimum SP estimate (greedy choic

Store V–S in priority queue Q.

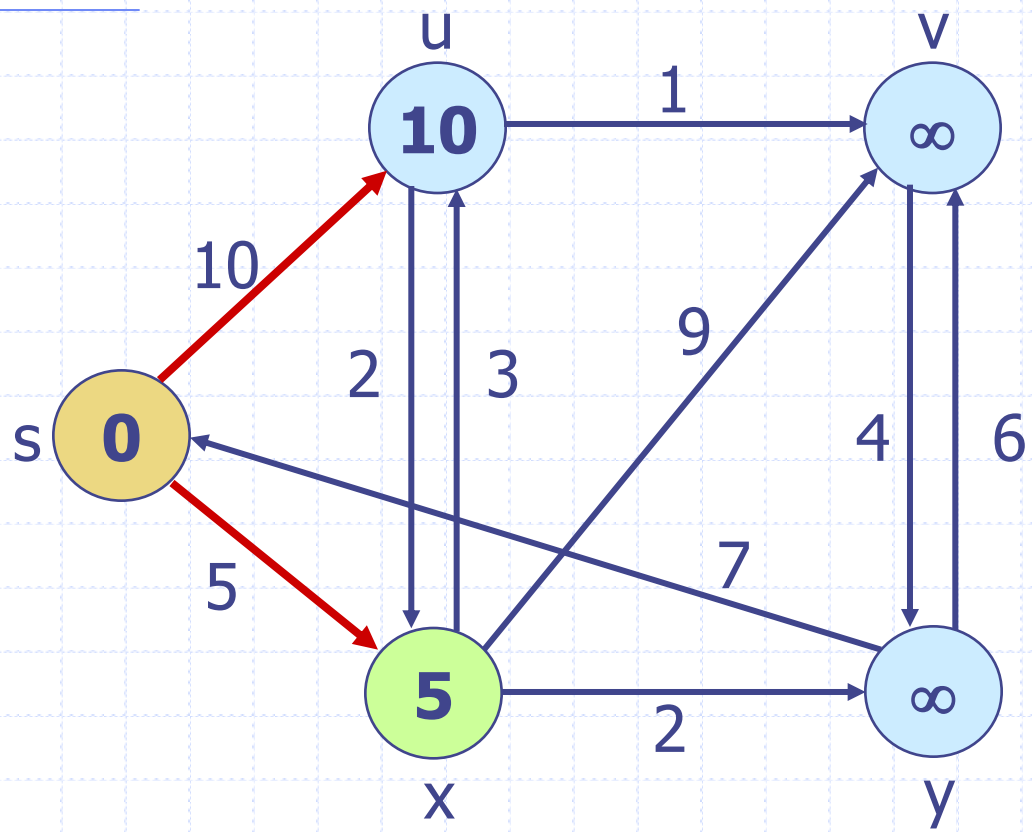
```
Initialize(G, s);
For each vertex v := V[G];
Do d[v]:= ∅
Pi[v]:=NIL

Relax(u,v,w)
if d[v]>d[u] + w[u,v];
then d[v]:=d[u]+w[u,v]
Pi[v]:=u
```

```
Initialize(G, s);
S := ∅;
Q := V[G];
while Q ≠ ∅ do
  u := Extract-Min(Q);
  S := S ∪ {u};
for each v ∈ Adj[u] do
    Relax(u, v, w)
  od
od
```

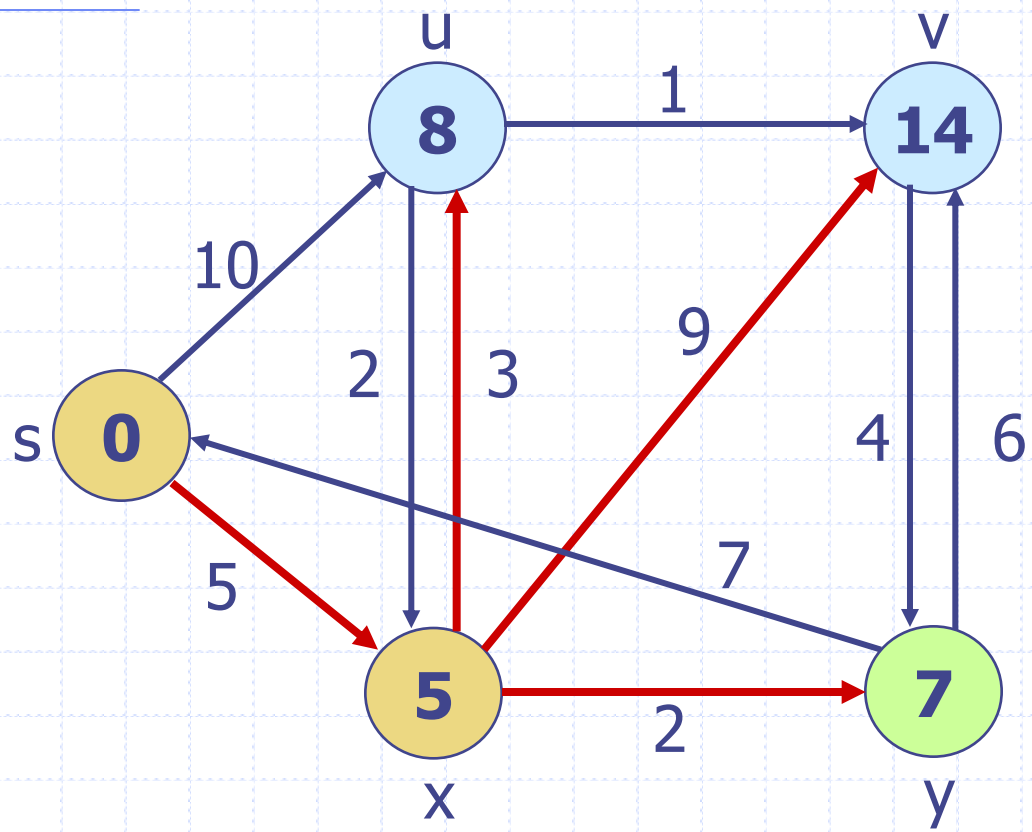
Example



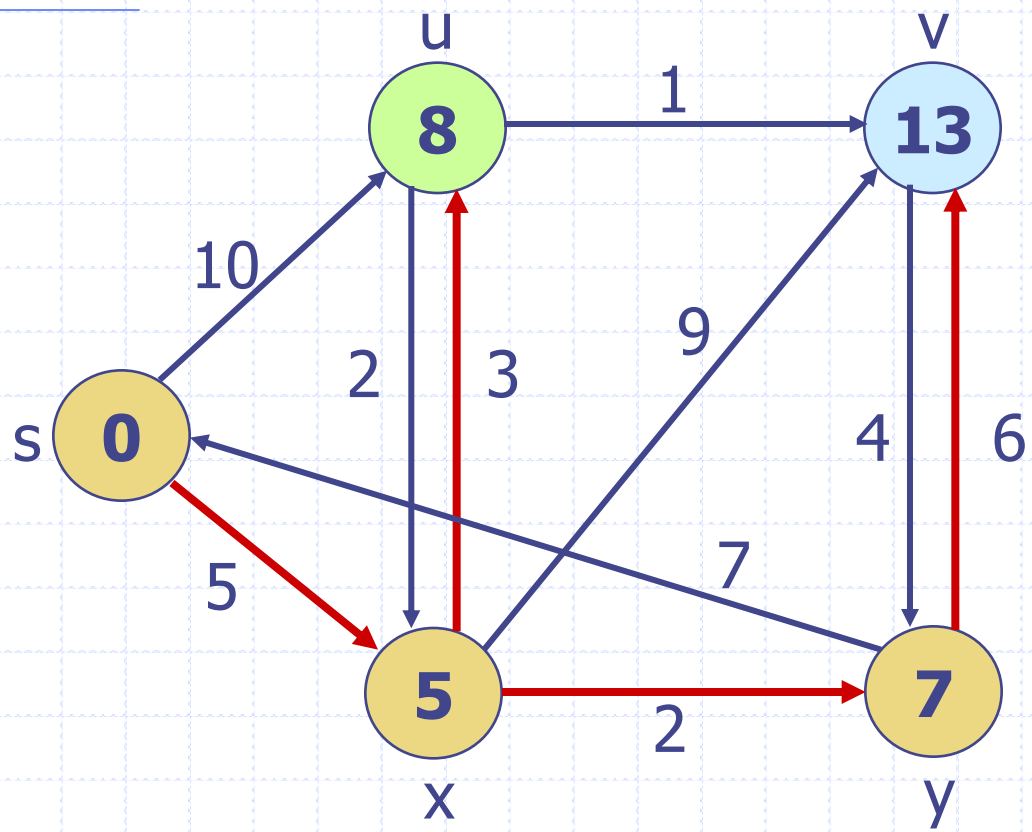
Example



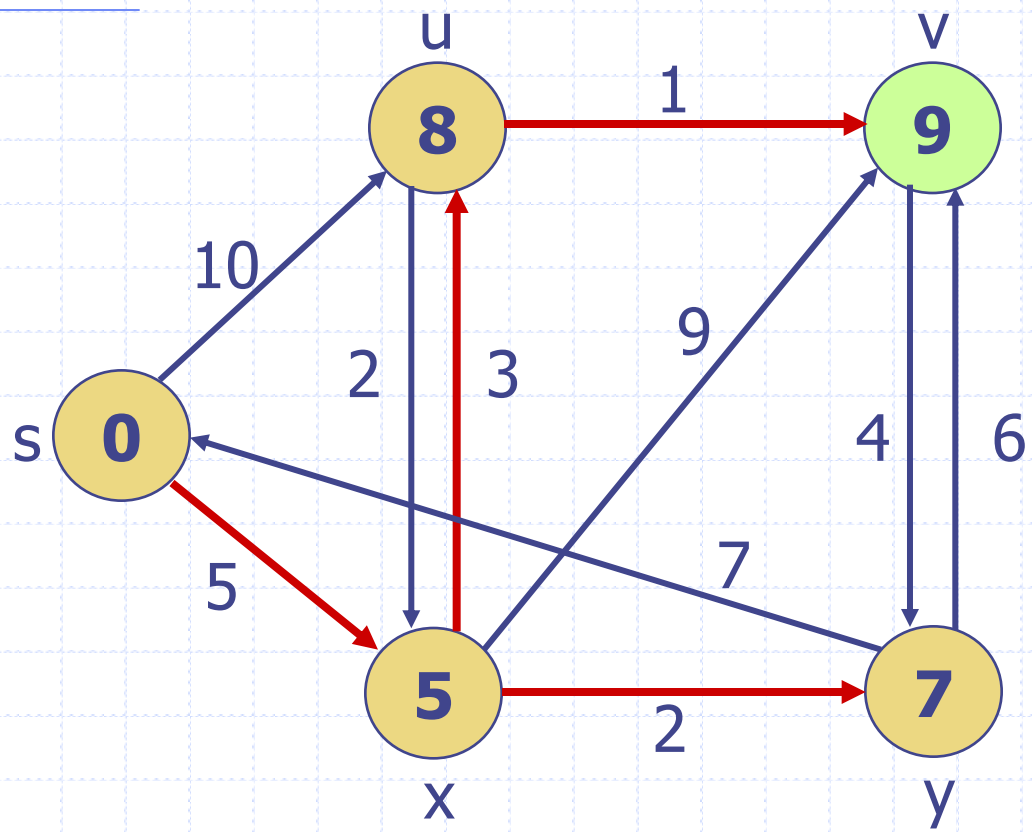
Example



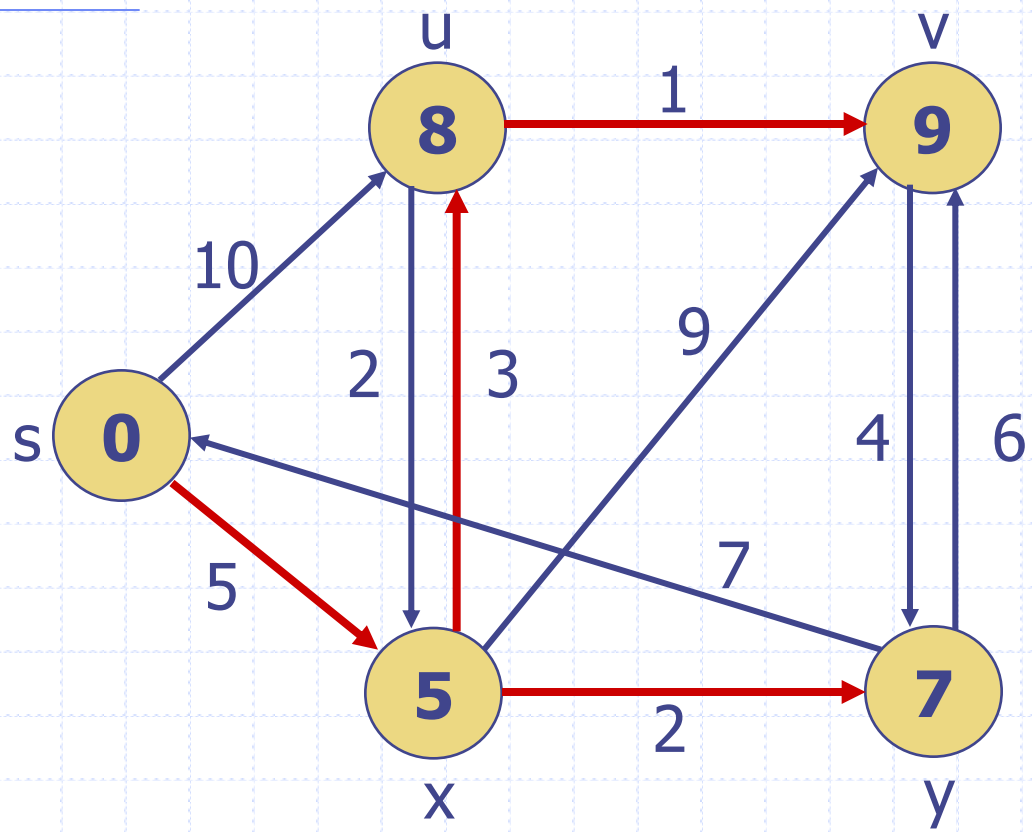
Example



Example



Example



Analysis of Dijkstra Algorithm

- The running time of Dijkstra algorithm depend on how the min-priority queue is implemented
- If we implement the min-priority queue with binary heap , then
  - Each EXTRACT-MIN operations takes O(lg V) time and there are |V| such operation.
  - Each DECREASE-KEY operation takes time O(lg V), & there are still at most |E| such operations.
  - There are the total running time is

$=O(V \lg V)+O(E \lg V)=O(E \lg V)$

Application

- Shortest path algorithms are applied to automatically find directions between physical locations, such as driving directions on websites like Google Map

Scope of Research

Computing Many-to-Many Shortest Paths Using Highway Hierarchie

Assignment

Q.1)What is single source shotest path?
Q.2)What is negative weight cycle in a graph?
Q.3)Explain Dijkstr algorithm with example.